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Optimal inventory and hedging decisions with CVaR consideration

Weili Xue [a], Lijun Ma [b,*], Houcai Shen [c]

[a] School of Economics and Management, Southeast University, China
[b] Department of Management Science, College of Management, Shenzhen University, China
[c] School of Management and Engineering, Nanjing University, China

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ABSTRACT

We consider a newsvendor who makes an ordering decision to meet stochastic demand and buys a put option written on the demand to hedge against the risk of low demand to maximize his expected utility, which is measured by the conditional value-at-risk (*CVaR*). The put option is fairly priced with specifications for the strike price and the strike quantity. With the consideration of lost-sale penalty cost, we derive structural results on the optimal ordering and hedging polices. We show that the newsvendor will not order more than that without the option contract when the strike quantity is predetermined and low, and he will order more when the strike quantity is a decision variable. Moreover, the optimal strike quantity is less than or equal to the optimal order quantity in the risk-neutral setting, and interestingly there are cases in which the optimal hedging ratio first increases, then keeps constant as the newsvendor becomes less risk averse. We also show that the value of the put option increases as the newsvendor becomes more risk averse and demand becomes more uncertain. Furthermore, the effect of risk aversion on the value of the option highly depends on the magnitudes of the system parameters.

1. Introduction

Short product life, long production lead time, low demand certainty, and high product variety are typical characteristics of today's business environment, which lead to low profit margin and high risk for firms. To survive in such an uncertain environment, firms have to adopt various approaches to cope with various business risks, especially the kind of demand risk that is associated with the weather. Examples of industries in which demand is highly correlated with weather are as follows: (1) a jacket company located in the North of England reported a third-quarter drop in earnings of 12% compared with the third-quarter of the previous year. It was also found that three years ago the third-quarter earnings were lower by as much as 14.5%. In both cases the company believed that the losses were due to milder than usual winters. Historical data indicate a 91% correlation between the temperature and the number of items of cold weather clothing sold by the company (Speedwell Weather Derivatives Ltd., 2003). (2) Demand for electricity is also weather sensitive. As shown in Fig. 1, electricity demand in California is highly correlated with the daily temperature where demand is strictly increasing after the threshold temperature (Franco and Sanstad, 2006). To hedge against the demand risk due to the non-financial impact of adverse (but non-catastrophic) weather conditions, weather contracts/derivatives (options, futures, combinations of both, etc.) have been widely used by firms such as the winter jacket manufacturer mentioned above. Zellner et al. (2001) report that in early 2001, a small clothing cataloger asked Enron to create a derivative product (i.e., a forward contract) to protect against hot weather that might decrease the sales of winter clothes. According to WRMA (2006), both exchanges and over-the-counter (OTC) markets for weather derivatives have emerged. Weather derivatives traded in the Chicago Mercantile Exchange (CME) have grown to a $45 billion per year industry in the U.S.

These above stories reveal the possibility and desirability that non-financial firms can diversify their demand risk through tradable buying/selling derivatives such as forward contracts or options, motivate us to investigate the optimal ordering and hedging strategy for a newsvendor when he wants to hedge his downside risk by a tradable put option. The put option gives the buyer "the right but not the obligation" to sell a quantity of the product at a given price after demand realization. Indeed, such a put option can be regarded as a simplified version of the put weather option for retailers whose demand is highly correlated with weather conditions. Moreover, for the ease of analysis, we

* Corresponding author. Tel.: +86 755 26535181; fax: +86 755 26534451.
E-mail addresses: wlxue1981@gmail.com (W. Xue), ljma@szu.edu.cn (L. Ma), hcshen@nju.edu.cn (H. Shen).

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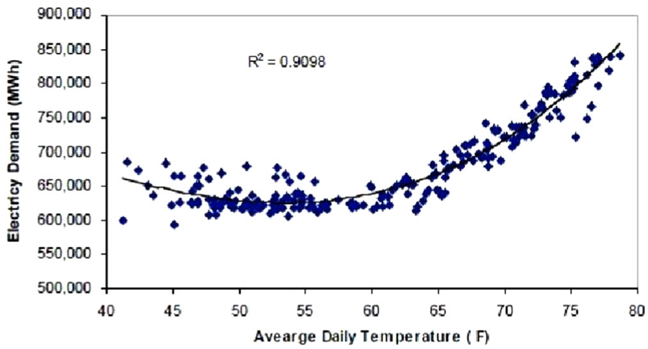

Fig. 1. Electricity demand with respect to average daily temperature.

assume that the option is written on the demand directly (see Gao et al., 2011). Such options have two important parameters, namely the strike quantity and the strike price. The strike quantity is the threshold level such that if the realized demand falls below it, the option writer pays the newsvendor the strike price for each leftover unit within the difference between the strike quantity and the realized demand; otherwise, the option writer does not pay anything and the option is valueless. In practice, the pricing of weather derivatives is usually by the non-arbitrage approach based on historical data or Monte Carlo based simulations (Garman et al., 2000) in which the non-financial variables such as “degree days” in weather contracts or “total demand” faced by the newsvendor can be determined by applying the actuarial approach to the analysis of historical data (Hull, 2003, p. 678).

Indeed, applying real options (i.e., combinations of spot markets, forward contracts, and options) to risk management in the high-tech industry, particularly with regard to the management of supply chains, has been generating significant value to shareholders in the long run (Billington et al., 2003). However, the tradable put option we studied in this paper is different to the real option contracts. First, the exercise of the proposed real put option depends on both the ordering quantity and the realized demand, while that of the tradable put option depends only on the realized demand. Second, the pricing strategies are different. The price of the real option is usually implicitly included in the ordering price, while the price of the tradable put option is determined by the non-arbitrage approach. Third, the risk transferring mechanisms are different. The tradable put option approach diversifies the supply chain risk to outsider risk bearer while the real option approach allocates the risk between the supply chain partners. Actually, the return policy offered by the supplier can also be regarded as a real option mechanism provided by the supplier. Thus, our put option is also different from the return policy in the above three ways.

Specifically, based on the newsvendor model as a framework, we develop the optimal ordering and hedging decisions for the newsvendor that is downside risk averse, when the put option written on demand is incorporated. Downside risk aversion implies that the decision maker cares more about under-performance relative to the mean, which is perceived as hazardous. Thus, considering a downside risk measure instead of the symmetric risk measure used in Chen and Parlar (2007) will provide more realistic results. Specifically, we use conditional value-at-risk (*CVaR*) to measure the newsvendor's risk attitude. The *CVaR* criterion is a coherent risk measure that measures the average value of the profit falling below a given quantile level, i.e., value-at-risk (*VaR*) (defined as the maximum profit at a specified confidence level Jorion, 2000). *CVaR* has emerged as a practical approach for modelling risk aversion with wide applications in economics, finance, and insurance (Rockafellar and Uryasev, 2000).

We make two major contributions in this paper. First, we derive structural results on the optimal ordering and put option decisions under the *CVaR* downside risk measure. Second, we examine how the system parameters, risk averse attitude, and demand uncertainty affect the value of the option. Our findings facilitate the implementation of put options in supply chain management. Specifically, we show that when the strike quantity is pre-determined and low, the newsvendor will not order more than that without the option contract because the benefit of the put option is totally offset by the cost of the put option. However, when the strike quantity is a decision variable, we find that, when an option is used, the optimal order quantity is higher than that without an option. Moreover, we find that the optimal strike quantity is less than or equal to the optimal order quantity in the risk neutral setting, and there are cases in which the optimal hedging ratio (i.e., strike quantity/order quantity) first increases, then keeps constant as the newsvendor is less risk averse, which is rather counterintuitive. Furthermore, we find that the value of the put option increases as the newsvendor becomes more risk averse and demand becomes more uncertain, and the effect of risk aversion on the value of the option highly depends on the magnitudes of the system parameters.

The rest of this paper is organized as follows: In the next section we review the related literature, which is followed by model formulation in Section 3. In Section 4 we solve the considered problem and generate insights from the analysis. In Section 5 we report the results of numerical examples to verify the theoretical analysis in Section 4. We conclude the paper and suggest topics for future research in Section 6.

2. Literature review

There are three research areas that are most relevant to our study, namely risk analysis in operations management, risk measurement, and supply chain contracts with options.

The literature on risk-averse operational models is quite limited. Lau (1980) analyzes the classical newsvendor model with respect to two different objective functions: maximization of the decision maker's expected utility and maximization of the probability of achieving a certain profit level. Eeckhoudt et al. (1995) study the effects of risk and risk aversion on a newsvendor's decisions when risk is measured by expected utility functions. Chen and Federgruen (2001) establish some standard infinite horizon inventory models to study the mean–variance tradeoff between customer waiting time and inventory level. Chen et al. (2007) consider a finite horizon inventory problem with exponential utility functions. They derive the optimal inventory decision, as well as the optimal pricing behaviour. Wu et al. (2010) study a commitment-option supply contract in the *CVaR* framework without information updating. Xu and Li (2010) study the newsvendor problem in the mean-*CVaR* framework. Buzacott et al. (2011) study a class of commitment-option supply contracts in the mean–variance framework. Ma et al. (2012a) study the channel bargaining problem with a risk-averse retailer who uses *CVaR* as the risk measure. Zhou et al. (2008) study the optimal ordering decisions for the multi-product problem with stochastic demand under return-*CVaR* model. They show that return-*CVaR* model is more flexible than the classical *CVaR* model. Chen et al. (2009) study the newsvendor problem with pricing and ordering decisions in the *CVaR* framework. They provide the conditions under which there exist optimal pricing and ordering decisions for the additive demand and multiplicative demand models. Caliskan-Demirag et al. (2011) compare the retailer rebate and customer rebate under *CVaR* risk measure. They find that risk attitude is an important parameter to determine which rebate scheme to use. Qiu et al. (2014) study the robust inventory decision under distribution uncertainty with a *CVaR*-based optimization approach, where demand information is incomplete. They find that the performance under both ellipsoid and box uncertain distributions is robust. For more recent literature on risk-averse operational models, we refer readers to Choi and Chiu (2012) for a thorough review. However, all the above works only use the option concept in the model, especially when the option price is provided through a fair price mechanism. For example, the commitment-option supply contracts studied in Wu et al. (2010) are actually flexible supply contracts.

The second relevant stream of research concerns risk measurement. It is well known that three major risk measures are widely used in financial studies, i.e., mean–variance and its variants, *VaR*, and *CVaR*. Each of these measures has its strengths and limitations. Mean–variance analysis is an important approach to modelling risk aversion (see Markowitz, 1959). It satisfies a class of decision makers with concave quadratic utility functions, but it is inadequate in the sense that it equally quantifies desirable upside outcomes and undesirable downside outcomes, rendering mean–variance a symmetric risk measure. *VaR* allows the decision maker to specify a confidence level for attaining a certain level of wealth (see Jorion, 2000). It is widely used to characterize downside risk in financial institutions. However, it has been criticized recent years in three aspects (Zhu and Fukushima, 2009): none sub-additivity under general distributions, existence of multiple local extrema for some discrete distributions, and inadequate characterization of uncertainty. To remedy the deficiencies of *VaR*, the *CVaR* measure, which measures the average profit falling below a given quantile level, was introduced. *CVaR* is a coherent risk measure that satisfies the properties of convexity and subadditivity (see Follmer and Schied, 2010). Schweitzer and Cachon (2000) present several alternative risk measures, including the prospect utility function, for the newsvendor model. Wang and Webster (2009) study the newsvendor problem under the loss-averse utility function with zero reference target. Ma (2008) extends their work to include a general reference target. Ma et al. (2012b) study the loss-averse newsvendor model with two ordering opportunities and market information updating. Although consideration of downside risk is very important, there is no previous work that incorporates *CVaR* into the newsvendor model with a put option.

The last relevant stream of literature is supply chain contracts with options. This stream of literature is quite limited. de Albeniz et al. (2006) consider the impact of a supply option contract on the newsvendor. Ding et al. (2007) study the interaction of operational and financial hedging policies of a risk-averse global firm facing demand and exchange uncertainty in the two-stage newsvendor setting. Gaur and Seshadri (2005) consider the problem of hedging against inventory risk in the newsvendor setting in which the product demand is correlated with the price of a tradable financial asset. Burnetas and Ritchken (2005) investigate the pricing of options with a downward sloping demand curve where a manufacturer offers the retailer the right to re-order (call option) and/or the right to return unsold goods at a pre-determined salvage value (put option). They formulate the problem to maximize the manufacturer's net present value and conclude that the retailer will either benefit from or be worse off with the options in terms of net present value. Wang et al. (2012) analyze the risk associated with introducing a call option in the two-period newsvendor setting. They find that even if providing a higher expected profit at the beginning of a planning horizon, supply contracts with options may have the risk associated with a worse performance later compared with the traditional newsvendor contract model. They also derive two important parameters for the buyer to estimate the risks of introducing options. Zhao et al. (2013) implement the value-based approach to price the real supply chain options and they show that their pricing schemes are more objective and fair than the traditional Stackelberg game approach. Chen and Parlar (2007) study the value of a put option based on the newsvendor model where the newsvendor uses a quadratic utility function and a put option can be purchased to reduce the loss resulting from low demand. The option is priced fairly with specifications on the strike quantity and the strike price. The newsvendor not only chooses the order quantity but also determines the strike price and/or the strike quantity of the put option. Recently, Gao et al. (2011) study the joint optimal ordering and weather hedging decisions with a mean-*CVaR* model. Modelling the demand as a stochastic decreasing function of the temperature index, they use the temperature call option to hedge against the demand risk associated with the temperature volatility. They find that the weather derivative hedging can increase the order quantity. In summary, none of the above research studies the joint decisions of inventory and put options in the newsvendor setting, except Chen and Parlar (2007) and Gao et al. (2011).

Our research is mostly related to Chen and Parlar (2007). With a risk-neutral objective and a quadratic utility function, they show that the same order quantity maximizes the expected profit with or without the option while the decisions on the strike price, as well as the strike quantity, of the put option do not affect the expected profit but the expected variance. Different from their work, our paper employs a downside risk measure, i.e., *CVaR*, to study the optimal ordering decision, as well as the optimal strike quantity for the put option, and analyze the effects of the system parameters on the choice of the put option. Moreover, in Chen and Parlar (2007), the decisions are made sequentially in the sense that the optimal hedging decision seeks to minimize the variance. We find that the structural results on the ordering and hedging quantities are related to the risk aversion attitude of the newsvendor and the strike price of the option. Furthermore, the inclusion of a put option does not necessarily induce the newsvendor to order more, as that depends on whether the strike quantity is a decision. This result is different from that of Gao et al. (2011), who find that weather derivative hedging always increases the order quantity.

3. Model formulation

3.1. The basic model

We consider a two-stage supply chain with a manufacturer and a retailer (the newsvendor). The newsvendor is risk averse and sells a fashion product, during a single selling season, at price s with random demand X. The retailer tries to maximize his risk measure, which is measured by the *CVaR* of his profit. *CVaR*, a downside risk measure, measures the average profit falling below a given quantile level (see Appendix for a further discussion of *CVaR*).

In addition to *CVaR*, mean–variance and *VaR* are two widely used risk measures in practice. The mean–variance risk measure equally quantifies desirable upside outcomes and undesirable downside outcomes. Yet, in practice, people care more about the undesirable downside outcomes. The *VaR* risk measure focuses on the undesirable downside outcomes. However, it is not a coherent risk measure due to its none sub-additivity under general distribution. The *CVaR* risk measure indeed remedies the deficiencies of mean–variance and *VaR* as it only considers the undesirable downside outcomes and is a coherent risk measure. Thus, we will employ *CVaR* as the risk measure throughout the whole paper.

At the beginning of the selling season, the newsvendor determines the order quantity Q at unit wholesale price c offered by the manufacturer. The cumulative distribution function and the probability distribution function of X are $F(x)$ and $f(x)$, with $x \in [0, \infty)$, respectively. To hedge against the risk of low demand in the selling season, the newsvendor can buy a put option from an option writer at price p. The put option specifies the strike price K_p and the strike quantity K_q before demand is realized, i.e., the newsvendor can exercise the option at strike price K_p at the end of the selling season when $x \le K_q$, while the put option has no value to the newsvendor when $x > K_q$. Thus, the option price p should be a function of both the strike price K_p and the strike quantity K_q. Such a put option may seem artificial at first glance. However, as discussed earlier, retailers in industries whose demand is highly correlated with some measurable and tradable variables, e.g., the weather index or the stock index, can use the corresponding financial derivatives to hedge against the demand risk. Furthermore, the findings of studies in such industries will provide insights and guidance to suppliers and retailers to hedge against demand risk in other industries.

In this paper we assume that the strike price K_p is exogenously determined, while the strike quantity K_q is determined by the newsvendor when he decides to engage in such an option contract. This is exactly the case with the weather derivative market in which the participants can determine the strike temperature (which is highly correlated with the demand for some industries), while the strike price of each degree of the temperature is pre-determined. In the following analysis, we denote p by $P(K_q)$ to reflect the dependence of the option price p on the strike quantity K_q, but omit the explicit relationship with K_p in the formula as we assume that K_p is exogenously determined.

Upon demand realization, the newsvendor can choose whether or not to exercise the put option (if any). In addition, the newsvendor incurs a unit shortage cost b for any unsatisfied demand and receives a unit salvage value v for any leftover inventory after demand is realized. We assume that the option writer can also salvage the product (if any) at unit value v at the end of the selling season. This assumption is mainly to ease the analysis without loss of generality. Actually, charging different salvage value for the newsvendor and the option writer does not change the main conclusions in this paper, yet complicates the analysis. On the other hand, this assumption has also its own practical background. For example, when the product has a public secondary market, which can be freely accessed by both the retailer and the supplier, the salvage prices will be the same. Even if there is no such market, it is also reasonable to assume the same salvage prices, as the prices for the same products will converge eventually by the spirit of non-arbitrage approach. Issues related to unsold units of the product, such as transshipment, re-selling, and so on are beyond the scope of our study here. Throughout this paper we assume that both parties have complete information on the unit revenues, unit costs, and the distribution of the random demand. To avoid triviality, we assume that $s>c>v$, $s \geq K_p \geq v$, which is consistent with Chen and Parlar (2007).

Thus, based on the definition of *CVaR* (see Appendix), the objective of the newsvendor, as a rational decision maker, is to choose the optimal nonnegative order and strike quantities Q and K_q, respectively, that solve the following optimization problem:

$$\max_{Q \geq 0, K_q \geq 0} \max_{\xi \in \mathbb{R}} \{g(Q, K_q, \xi)\}. \tag{1}$$

in which

$$g(Q, K_q, \xi) := \xi - \frac{1}{\eta} \mathrm{E}[\xi - \hat{\pi}(X; Q, K_q)]^+,$$

and $\hat{\pi}(x; Q, K_q)$ is the newsvendor's profit, with the put option, as a function of the order quantity, strike quantity and the realized demand x at the end of the selling season.

4. Static inventory and hedging decisions with a put option

4.1. Price of the put option

Now we analyze the price $P(K_q)$ of the put option as a function of K_q. In the finance literature, the option price of an option is always priced by its expected discounted payoff evaluated under the risk-neutral measure (i.e., by the no-arbitrage approach). However, if the underlying asset is not tradable, we cannot price the put option by the no-arbitrage approach but by the actuarial approach (Hull, 2003, p. 678). Based on this idea, in this paper, like Chen and Parlar (2007), we assume that the option writer charges the newsvendor a positive risk premium r plus the expected benefit accruing to the newsvendor so that the price of the option is obtained as follows:

$$P(K_q) = (K_p - v) \int_0^{K_q} (K_q - x) f(x)\, dx + r, \tag{2}$$

where

$$\frac{\partial P}{\partial K_q} = (K_p - v) F(K_q) \geq 0, \tag{3}$$

$$\frac{\partial^2 P}{\partial K_q^2} = (K_p - v) f(K_q) \geq 0. \tag{4}$$

Here r measures both the relative risk attitude of the option writer and the additional cost incurred by the newsvendor when he transfers his demand risk to the third party. Furthermore, by comparing the case where there is no put option with the case where there is a put option, r can be used to analyze whether the newsvendor should or should not buy the put option. Furthermore, we can determine the break-even value of r. In our analysis, we assume that r is a constant, which is consistent with Chen and Parlar (2007). For further discussion of the valuation of this option and the risk premium r, we refer the reader to Chen and Parlar (2007).

Note that our pricing formula for the put option contract (2) leads to the fact that the price of the put option is endogenously determined by the newsvendor in choosing the strike quantity K_q. This is one of the major differences between our study and Wu et al. (2010). In their work, the (call) option contract's price and the strike quantity are both determined exogenously. In contrast, we show that, by (2) in our model, the newsvendor's decision on the strike quantity K_q also affects the price of the option. In fact, the price of the option is convex increasing in the strike quantity.

4.2. Newsvendor's profit

Now, we go further to discuss the formulation of the newsvendor's profit. When $K_q \leq Q$, the expression of $\hat{\pi}(x; Q, K_q)$ for a realized value of demand $X=x$ is

$$\hat{\pi}(x; Q, K_q) = \begin{cases} \hat{\pi}_1(x; Q, K_q) & \text{if } x < K_q, \\ \hat{\pi}_2(x; Q, K_q) & \text{if } K_q \leq x \leq Q, \\ \hat{\pi}_3(x; Q, K_q) & \text{if } x > Q, \end{cases} \tag{5}$$

where

$$\hat{\pi}_1(x; Q, K_q) = sx + K_p(K_q - x) + (v - c)Q - vK_q - P(K_q),$$
$$\hat{\pi}_2(x; Q, K_q) = (s - v)x + (v - c)Q - P(K_q),$$
$$\hat{\pi}_3(x; Q, K_q) = (s - c + b)Q - bx - P(K_q).$$

Here, if the realized demand $x < K_q$, then the newsvendor makes a profit of $s-c$ for each of the x units he sells, receives $K_p - c$ for the $K_q - x$ units that are covered by the put option, and receives a net amount of $v-c$ for the $Q-K_q$ units that are not covered by the put option but salvaged. When $K_q \leq x \leq Q$, the option cannot be exercised and the newsvendor receives a net amount of $v-c$ for the $Q-x$ units that are salvaged. Finally, when $x>Q$, the newsvendor can sell only Q units and incurs a goodwill loss of b per unit for $x-Q$ units of shortage. In particular, when $K_q=0$ and $P(K_q)=0$, $\hat{\pi}(x; Q, K_q)$ in (5) reduces to the classical newsvendor problem. It is well known that the unique solution to this newsvendor problem in the risk neutral setting Q^u (where the superscript u denotes "risk neutral") solves

$$F(Q^u) = \frac{s - c + b}{s - v + b}.$$

We see in the subsequent analysis that Q^u plays an important role in determining the optimal strike quantity of the put option.

When $K_q > Q$, the expression of $\hat{\pi}(x;Q,K_q)$ for a realized value of demand $X=x$ reduces to

$$\hat{\pi}(x;Q,K_q) = \begin{cases} \hat{\pi}'_1(x;Q,K_q) & \text{if } x < Q, \\ \hat{\pi}'_2(x;Q,K_q) & \text{if } x > Q, \end{cases} \tag{6}$$

where

$$\hat{\pi}'_1(x;Q,K_q) = sx + K_p(Q-x) - cQ - P(K_q),$$
$$\hat{\pi}'_2(x;Q,K_q) = (s-c+b)Q - bx - P(K_q).$$

Regardless of the price of the put option, i.e., $P(K_q)=0$, the expression $\hat{\pi}(x;Q,K_q)$ in this case is equivalent to the newsvendor problem with a salvage value K_p.

Before we further analyze our model, we first present the optimal inventory policy for a risk-averse newsvendor to maximize his *CVaR* when there is no put option. Denote Q^N as the order quantity without any put options and Q^A as the order quantity with a put option both under the risk-averse setting. Xu and Li (2010) derived the result, which we summarize in the following lemma.

Lemma 4.1. *The optimal order quantity for the risk-averse newsvendor is*

$$Q^N = \frac{1}{s+b-v}\left[(s-v)F^{-1}\left(\frac{\eta(s+b-c)}{s+b-v}\right) + bF^{-1}\left(1-\frac{\eta(c-v)}{s+b-v}\right)\right].$$

As discussed in Xu and Li (2010), Q^N may not be monotonic in η, and may be less or greater than the corresponding risk-neutral solution Q^u. That is, the optimal order quantity under the *CVaR* criterion may be higher or lower than its counterpart under the risk-neutral criterion. Moreover, Xu and Li (2010) showed that Q^N is strictly increasing in b, while it is strictly decreasing in c.

4.3. Optimal ordering decision with given strike quantity

We now first consider the optimal ordering policy for the newsvendor with a given strike quantity K_q. We will analyze the case where the newsvendor can determine both the strike quantity and the ordering quantity later.

To facilitate our analysis, we define the following values:

$$\overline{K} = \frac{b}{s+b-K_p}F^{-1}\left(1-\frac{\eta(c-K_p)}{s+b-K_p}\right) + \frac{(s-K_p)}{s+b-K_p}F^{-1}\left(\frac{\eta(s+b-c)}{s+b-K_p}\right),$$
$$K^M = \frac{b}{s+b-K_p}F^{-1}\left(1-\frac{\eta(c-v)}{s+b-v}\right) + \frac{(s-K_p)}{s+b-K_p}F^{-1}\left(\frac{\eta(s+b-c)}{s+b-v}\right),$$
$$\underline{K} = F^{-1}\left(\eta\frac{s+b-c}{s+b-v}\right),$$

where $\overline{K}$ is defined when $K_p < c$, which is the optimal solution for the *CVaR* maximizer with unit salvage value K_p when compared with Q^N in Lemma 4.1. Note that when the shortage cost $b \to 0$, $\underline{K}$ is equal to Q^N, so $\underline{K}$ is a critical value for the order quantity under *CVaR* when shortage cost is negligible. K^M is a linearly weighted combination of $\overline{K}$ and $\underline{K}$. Similar to Xu and Li (2010), we have the following lemma about the properties of these values.

Lemma 4.2. *$\overline{K}$ and K^M are increasing in K_p and b, and decreasing in c; K^M is increasing in v.*

We omit the proof as the results can be easily obtained by taking the first-order derivatives. From this lemma and the assumption that $K_p \geq v$, we see that $\overline{K}$ and K^M are always greater than Q^N, and $\overline{K}$ is greater than K^M. On the other hand, as $\eta \leq 1$ by the definition of *CVaR*,

$$1 - \frac{\eta(c-v)}{s+b-v} - \frac{\eta(s+b-c)}{s+b-v} \geq 0,$$

so

$$\begin{aligned} Q^N &= \frac{1}{s+b-v}\left[(s-v)F^{-1}\left(\frac{\eta(s+b-c)}{s+b-v}\right) + bF^{-1}\left(1-\frac{\eta(c-v)}{s+b-v}\right)\right] \\ &\geq \frac{1}{s+b-v}\left[(s-v)F^{-1}\left(\frac{\eta(s+b-c)}{s+b-v}\right) + bF^{-1}\left(\frac{\eta(s+b-c)}{s+b-v}\right)\right] = \underline{K}. \end{aligned}$$

The following lemma summarizes the above analysis.

Lemma 4.3. $\overline{K} \geq K^M \geq Q^N \geq \underline{K}$.

In fact, these values and their corresponding properties define different areas of the strike quantity K_q in which the optimal order quantity varies, as stated in the following theorem.

Theorem 4.1. *Given the strike quantity K_q, if the strike price is larger than the ordering cost, i.e., $K_p \geq c$, the optimal ordering quantity Q^A is*

$$Q^A = \begin{cases} K_q, & K_q \geq K^M, \\ \frac{1}{s-v+b}[(K_p-v)K_q + (s+b-K_p)K^M], & K^M > K_q \geq \underline{K}, \\ Q^N, & \underline{K} > K_q, \end{cases}$$

if the strike price is lower than the ordering cost, i.e., $v \leq K_p < c$, the optimal ordering quantity Q^A is

$$Q^A = \begin{cases} \overline{K}, & K_q \geq \overline{K}, \\ K_q, & \overline{K} > K_q \geq K^M, \\ \frac{1}{s-v+b}[(K_p-v)K_q + (s+b-K_p)K^M], & K^M > K_q \geq \underline{K}, \\ Q^N, & \underline{K} > K_q. \end{cases}$$

An illustration of the optimal solution is shown in Fig. 2. If the ordering cost c is lower than the strike price K_p, the optimal order quantity depends on the relative value of K_q with respect to $\underline{K}$ and K^M: when $K_q \geq K^M$, it is optimal for the newsvendor to order

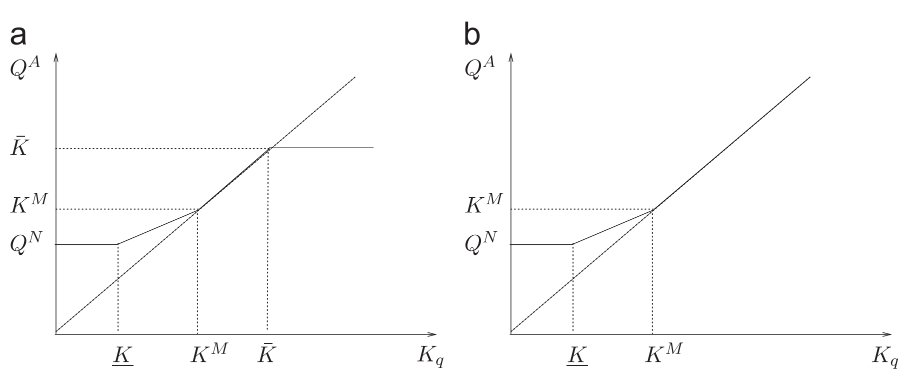

Fig. 2. Optimal order quantity when the strike quantity of the put option is given. (a) The case for $K_p < c$. (b) The case for $K_p \geq c$.

exactly the same quantity as the strike quantity K_q; when $K_q \in [\underline{K}, K^M)$, the newsvendor should order the quantity that is a linear combination of K_q and K^M, i.e., $Q^A = \lambda K_q + (1-\lambda)K^M$ with $\lambda = K_p/(s-v+b)$; when K_q is lower than $\underline{K}$, the newsvendor should only order Q^N, which is the quantity when there is no put option. If the ordering cost c is higher than the strike price K_p, the optimal ordering policy remains the same when $K_q < K^M$. However, when $K_q \geq K^M$, we can further divide K_q into two groups by $\overline{K}$: when $K_q \geq \overline{K}$, it is optimal to order exactly $\overline{K}$; when $K_q \in [K^M, \overline{K})$, it is optimal to order K_q.

Combining this theorem with Lemma 4.3, we show that the optimal order quantity Q^A is lower than K_q only if $K_p < c$; otherwise, it is always optimal for the newsvendor to order more than (or equal to) the strike quantity. Moreover, when K_q is larger than $\underline{K}$, the optimal order quantity will be higher than that when there is no put option. This is rather intuitive as the put option can be used to hedge against demand risk. However, when K_q is lower than $\underline{K}$, the optimal order quantity is equal to that when there is no put option. This implies that the inclusion of a put option does not increase the order quantity when the strike quantity of the option is rather low. This result is somewhat counter-intuitive. The reason is that, in this case, the benefit from the option contract is totally offset by the additional cost incurred by the newsvendor.

4.4. Optimal decision on the strike quantity and order quantity

Based on the analysis in the previous section, we now analyze how the newsvendor should determine both the strike quantity and the order quantity to maximize his *CVaR*. We assume that $K_p \geq v$ and $\eta < 1$ to rule out the cases where the option contract is of no use and the newsvendor is risk neutral.

Before we present the optimal policy, we first define two critical values Q_1 and Q_2 as follows:

$$Q_1 = \frac{1}{s-v+b}[(K_p - v)Q^u + (s+b-K_p)K^M] \quad \text{and} \quad G(Q_2) = 0,$$

in which

$$G(t) = (s+b-K_p)t - bF^{-1}\left(1 - \eta\frac{(K_p-v)F(t)-(K_p-c)}{s+b-K_p}\right) - (s-K_p)F^{-1} \times \left(\eta\frac{(s+b-c)-(K_p-v)F(t)}{s+b-K_p}\right).$$

Here Q_1 is a linear combination of the risk-neutral quantity Q^u and the threshold K^M, while Q_2, if exists, lies in $(F^{-1}((K_p-c)/(K_p-v)), F^{-1}((s+b-c)/(K_p-v))$, in which the function $F^{-1}(\cdot)$ is well defined. We will discuss the existence of Q_2 later. We present the following lemma about the properties of Q_1 and Q_2 (when exists).

Lemma 4.4. *Both Q_1 and Q_2 are decreasing in the unit cost c, and are increasing in the penalty cost b, strike price K_p, and salvage value v.*

We omit the proof, which is straightforward by taking first-order derivatives. The result establishes monotone properties of Q_1 and Q_2 with respect to the system parameters, which are useful to account for the monotone properties of the optimal ordering and hedging policies in later analysis.

Similar to the discussion in Xu and Li (2010), the relative sizes of Q_1 and Q^u depend on the system parameters. For example, when $K_p = s$, $Q_1 > Q^u$, whereas when $K_p = v$, $Q_1 = Q^N$, which is lower than Q^u as η approaching 1. This is also illustrated in Fig. 3(a). Thus, the value of Q_1 may be less or greater than the risk-neutral value Q^u, as η varies. Moreover, we can establish the following result concerning the relationships among Q_1, Q_2, and Q^u.

Lemma 4.5. *The relationships among Q_1, Q_2, and Q^u are one of the following: (i) $Q_1 < Q^u$ and $K^M < Q_2 < Q^u$; (ii) $Q_1 > Q^u$ and $Q_2 > Q^u$; and (iii) $Q_1 = Q^u = Q_2$.*

This lemma means that both Q_1 and Q_2 are higher, lower, or equal to Q^u at the same time (see Fig. 3(a)). This property will assist our analysis of the existence of Q_2 when $Q_1 < Q^u$ for $b > 0$ and $s > K_p$. To see this, substituting Q^u into $G(Q)$ yields $G(Q^u) = (s+b-K_p)(Q^u - Q_1) > 0$, while $\lim_{Q \to F^{-1}((K_p-c)/(K_p-v))} G(Q) < 0$. However, for practical and numerical analysis purposes, the definition area of $F(x)$ is always restricted to the finite area $[\underline{U}, \overline{U}]$ with $\underline{U} \geq 0$. In this case, we find that Q_2 exists when

$$\overline{U} \geq \frac{s+b-K_p}{b}F^{-1}\left(\frac{K_p-c}{K_p-v}\right) - \frac{s-K_p}{b}F^{-1}(\eta)$$

or $K_p \leq c$. In this paper we assume that $F(x)$ is continuous and differentiable when $x \in [0, \infty)$. Moreover, this lemma is essential to further analysis of the optimal policy, which is stated in the following theorem.

Theorem 4.2. *The optimal order quantity Q^* and the optimal strike quantity K_q^* for the newsvendor are*

$$(Q^*, K_q^*) = \begin{cases} (Q_1, Q^u) & \text{if } Q_1 \geq Q^u; \\ (Q_2, Q_2) & \text{otherwise}. \end{cases}$$

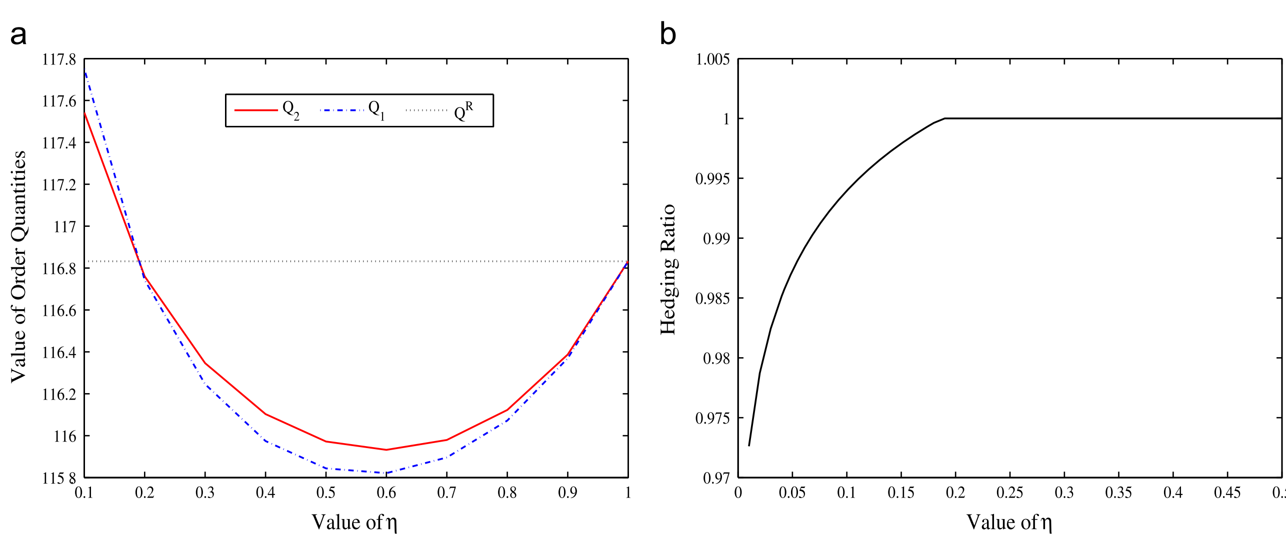

Fig. 3. An illustration of the optimal policies with respect to η (with parameters in Table 1). (a) Optimal Q_1 and Q_2 with respect to η. (b) Optimal hedging ratio ρ with respect to η.

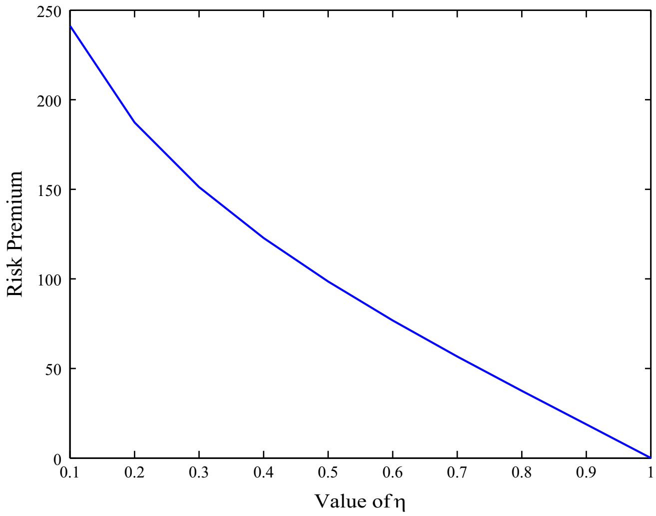

Fig. 4. Break-even risk premium $\bar{r}$ with respect to η (with parameters in Table 1).

Thus, we have derived the optimal policy for the newsvendor to maximize his *CVaR*: when the system parameters (i.e., s, c, R, v, b, and η) satisfy $Q_1 \geq Q^u$, the optimal order quantity is Q_1, while the optimal strike quantity is equal to the risk-neutral newsvendor solution Q^u; when these parameters satisfy $Q_1 < Q^u$, the optimal order quantity and strike quantity are equal to the same value Q_2. This result is different from that of the analysis in the symmetric quadratic concave function framework, in which the order quantity is always Q^u (see Chen and Parlar, 2007). However, from Lemma 4.5 and Theorem 4.2, we find that the inclusion of a put option for a downside risk-averse decision maker may increase, decrease, or equal to the risk-neutral order quantity.

Combining this theorem with Lemma 4.5, we conclude that the order quantity with a put option is higher than that without a put option, i.e., $Q^* > K^M > Q^N$. Thus, when the newsvendor can choose both the order quantity and the strike quantity, the inclusion of the put option always induces the newsvendor to order more. This is different from the case where the strike quantity is given exogenously, in which the newsvendor only orders more when the strike quantity is large. Moreover, it is interesting to note that the newsvendor's optimal strike quantity is lower than or equal to the risk-neutral newsvendor solution. Other properties associated with the policy parameters are straightforward: the optimal order quantity and the strike quantity are decreasing in the unit cost and the penalty cost, while they are increasing in the unit salvage value; and the optimal order quantity and strike quantity may not be monotone with risk aversion η (see Fig. 3(a)).

Remark. If we define the optimal hedging ratio as $\rho = K_q/Q$, then the above theorem implies that when it is optimal for the newsvendor to order more than the risk-neutral solution, the optimal hedging ratio $\rho < 1$; otherwise, $\rho = 1$. This is also illustrated in Fig. 3(b). This result shows that the hedging ratio, ρ, is increasing in η, i.e., high risk aversion implies a low hedging ratio. This is rather counter-intuitive at first glance. The reason lies in the fact that, in this case, $Q_1 > Q^u$ when η is small, which results in the maximum hedging quantity Q^u. Thus, the optimal hedging ratio, ρ, depends only on the value of Q_1, which is decreasing in η as Fig. 3(a) shows.

Remark. When $b = 0$, we have

$$Q_1 = \frac{s-K_p}{s-v} F^{-1}\left(\eta \frac{s-c}{s-v}\right) + \frac{K_p-v}{s-v} F^{-1}\left(\frac{s-c}{s-v}\right) < Q^u \tag{7}$$

$$Q_2 = F^{-1}\left(\frac{\eta(s-c)}{(s-K_p)+(K_p-v)\eta}\right). \tag{8}$$

Thus, when there is no shortage penalty, it is always to the newsvendor's advantage to hedge all his order quantity. Furthermore, Q_2 is increasing in η. That is, the optimal order quantity is lower when the newsvendor is more risk averse. This is similar to the observation by Wu et al. (2010). However, our analysis shows that this holds only when there is no goodwill cost, i.e., $b = 0$. On the other hand, when $b > 0$, the optimal order quantity does not increase in η (see Fig. 3(a)). This implies that when the strike price is equal to the selling price, the optimal order quantity with a put option increases with the risk averseness of the newsvendor.

4.5. Value of the put option

So far we have performed analysis under the assumption that the newsvendor will accept the put option. However, to buy a put option, the newsvendor will incur a cost $P(K_q)$, which may cancel out the benefit that the option brings. Thus, in this section, we further study the newsvendor's optimal decision as to whether or not to choose the put option.

Let CVaR_η^K, CVaR_η^N be the maximum *CVaR* with and without a put option, respectively. Then, the optimal decision for the newsvendor is

$$\begin{cases} \text{do not buy the option} & \text{if } \text{CVaR}_\eta^K < \text{CVaR}_\eta^N, \\ \text{buy the option} & \text{if } \text{CVaR}_\eta^K > \text{CVaR}_\eta^N. \end{cases}$$

If $\text{CVaR}_\eta^K = \text{CVaR}_\eta^N$, then the newsvendor chooses either decision arbitrarily.

To ease the notation, let

$$A = F^{-1}\left(1 - \frac{\eta(c-v)}{s+b-v}\right), \quad A' = F^{-1}\left(1 - \eta \frac{(K_p-v)F(Q_2)-(K_p-c)}{s+b-K_p}\right),$$

and

$$B' = F^{-1}\left(\eta \frac{(s+b-c)-(K_p-v)F(Q_2)}{s+b-K_p}\right).$$

We can calculate the optimal *CVaR* for the newsvendor when the option is used as follows: when $Q_1 \geq Q^u$,

$$\text{CVaR}_\eta^K = -r + (K_p - v)\int_0^{Q^u} xf(x)\,dx - \frac{1}{\eta}\left[b\int_A^\infty xf(x)\,dx - (s-K_p) \times \int_0^{\underline{K}} xf(x)\,dx\right],$$

and when $Q_1 < Q^u$, we have $A' > A$, $B' > \underline{K}$, and

$$\text{CVaR}_\eta^K = -r + (K_p - v)\int_0^{Q_2} xf(x)\,dx - \frac{1}{\eta}\left[b\int_{A'}^\infty xf(x)\,dx - (s-K_p) \times \int_0^{B'} xf(x)\,dx\right].$$

Thus, by comparing CVaR^K and CVaR^N, we have the following theorem about whether a put option is valuable to the newsvenodr and what the break-even value of the risk premium is, at which there is no difference for the newsvendor to choose a put option or not.

Table 1
Initial values for basic parameters.

Parameters	Values	Parameters	Values
μ	100	v	5
σ	20	η	0.5
c	12	b	10
s	20	K_p	15

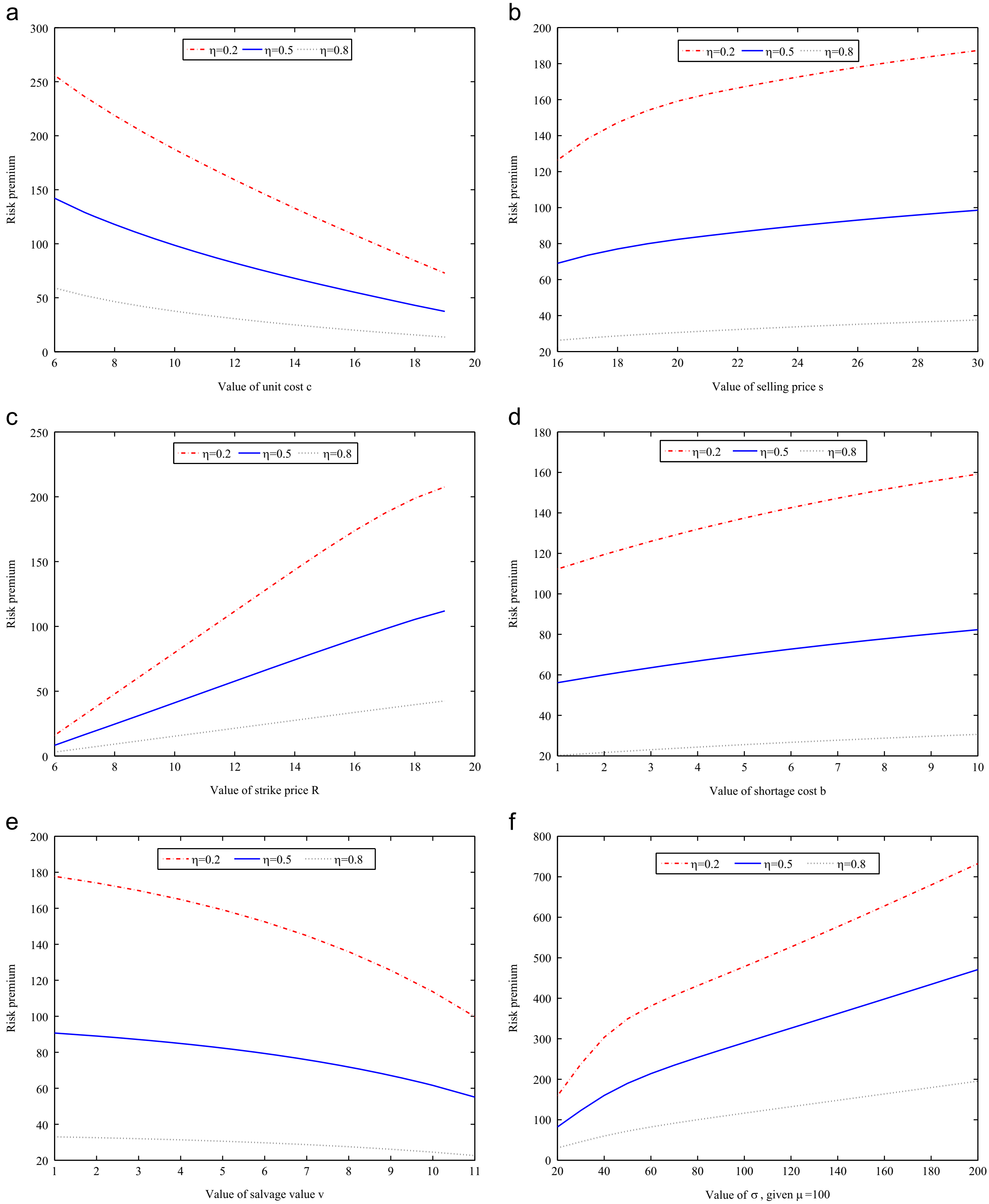

Fig. 5. An illustration of the break-even premium $\bar{r}$ with respect to cost parameters. (a) Break-even risk premium $\bar{r}$ with respect to c. (b) Break-even risk premium $\bar{r}$ with respect to s. (c) Break-even risk premium $\bar{r}$ with respect to K_p. (d) Break-even risk premium $\bar{r}$ with respect to b. (e) Break-even risk premium $\bar{r}$ with respect to v. (f) Break-even risk premium $\bar{r}$ with respect to σ.

Theorem 4.3. *When* $r=0$, $\mathrm{CVaR}_\eta^K \ge \mathrm{CVaR}_\eta^N$. *The corresponding break-even value* $\bar{r}$ *to use a put option is given as follows:*

(i) *if* $Q_1 \ge Q^u$, *we have*

$$\bar{r} = (K_p - v)\left[\int_0^{Q^u} xf(x)\,dx - \frac{1}{\eta}\int_0^{K} xf(x)\,dx\right],$$

(ii) *if* $Q_1 < Q^u$, *we have*

$$\bar{r} = (K_p - v)\int_0^{Q_2} xf(x)\,dx + \frac{1}{\eta}\left[b\int_A^{A'} xf(x)\,dx + (s-K_p)\int_0^{B'} xf(x)\,dx - (s-v)\int_0^{K} xf(x)\,dx\right].$$

That is, if the option writer charges a zero premium, it is always optimal for the newsvendor to use the put option, as the one without a put option is a special case with $K=0$. Otherwise, the newsvendor's decision as to whether or not to choose the put option depends on whether the premium is higher or lower than the break-even value $\bar{r}$. Moreover, we observe that $\bar{r}$ is decreasing in η (see Fig. 4), which implies that the more risk averse the newsvendor is, the higher is the break-even premium. The risk premium is thus compensated by the risk attitude of the newsvendor.

5. Numerical examples

In the previous section we established both analytically and numerically the monotone properties of the optimal order quantity with respect to the cost parameters. Moreover, we derived the formula of the risk premium threshold, below which it is optimal for the newsvendor to use the option contract. However, it is very challenging to establish the relationships among the optimal solution and demand uncertainty and risk attitude, especially for the break-even risk premium $\bar{r}$ under *CVaR*, which is also called the value of the put option. In this section we present some numerical examples to demonstrate the impacts of the risk attitude η, system parameters (unit cost, selling price etc.), and demand uncertainty on the break-even risk premium $\bar{r}$, respectively. The objective is threefold: (i) to compare the results at different level of risk aversion, (ii) to compare the results at different levels of demand uncertainty, and (iii) to demonstrate the impacts of system parameters on the break-even risk premium.

The numerical examples are based on a truncated normal distribution (i.e., the random variable defined in $[0,\infty)$) with mean μ and standard variance σ, which is used to measure the demand uncertainty. The values for the parameters of the basic model are listed in Table 1. In carrying out the computation, we varied one of the parameters (i.e., c, s, K_p, b, v, and σ) while holding the other parameters fixed.

Fig. 5 illustrates the relationships between the break-even risk premium $\bar{r}$ and c, s, K_p, b, v, and σ, for $\eta=0.2$, 0.5, and 0.8 respectively. Specifically, from Fig. 5(a), we see that the break-even risk premium $\bar{r}$ decreases as the unit cost increases. This is because the high unit cost shrinks the newsvendor's order quantity and strike quantity, which reduces the risk of the newsvendor, thus reducing the value of the put option. Following similar reasoning, Fig. 5(b) shows that $\bar{r}$ increases as the selling price increases and Fig. 5(d) shows that $\bar{r}$ increases as the shortage cost b increases. Fig. 5(c) and (e) show the effects of the strike price K_p and the salvage value v, both of which directly affect the value of the option. Fig. 5(c) shows that the value of the put option increases as the strike price increases, although the non-arbitrage cost of the put option (i.e., $P(K_q)$ for $r=0$) is increasing in K_p. Fig. 5(e) shows that the value of the put option increases as the salvage value decreases, although the non-arbitrage cost of the put option is decreasing in v. Demand uncertainty is also an important factor that affects the break-even risk premium, as shown in Fig. 5(f). The more uncertain the demand is, the greater is the break-even risk premium.

By comparing the distance between each line with $\eta = 0.2$, 0.5, and 0.8 in each figure in Fig. 5, we conclude that the effect of risk aversion on risk premium highly depends on the system parameters' magnitudes. Specifically, Fig. 5(a) shows that the effect of risk aversion η on $\bar{r}$ increases as c decreases, i.e., the effect of risk aversion on risk premium is low when the unit cost is high. Fig. 5(b) shows that the effect of risk aversion increases as the selling price increases. Fig. 5(d) shows the effect of risk aversion on risk premium is high when the shortage cost is high. Fig. 5(c) implies that the effect of risk aversion on risk premium is high when the strike price is high. Fig. 5(e) shows that the effect of risk aversion on risk premium is high when the salvage value is low. Fig. 5(f) shows that the effect of risk aversion on risk premium is high when demand uncertainty is large.

6. Conclusions

In this paper we develop the optimal ordering and hedging policies for a newsvendor that is downside risk averse when a put option written on demand is incorporated. The objective of the newsvendor is to maximize his utility, which is measured by *CVaR*. Based on the newsvendor framework, we derive structural results on the optimal ordering and put option decisions with the *CVaR* downside risk measure. Furthermore, we study how the system parameters, risk averse attitude, and demand uncertainty affect the value of the option. Our findings should facilitate the implementation of put options in supply chain management. Specifically, we show that when the strike quantity is pre-determined and low, the newsvendor will not order more than that without the option contract, as the benefit of the put option is totally offset by the cost of the put option. However, when the strike quantity is a decision variable, we find that, when an option is used, the optimal order quantity is higher than that without an option. Moreover, we find that the optimal strike quantity is less than or equal to the optimal order quantity in the risk neutral setting, and there exist cases in which the optimal hedging ratio first increases, then keeps constant as the newsvendor is less risk averse, which is rather counter-intuitive. Furthermore, we find that the value of the put option increases as the newsvendor becomes more risk averse and demand becomes more uncertain, and the effect of risk aversion on the value of the option highly depends on the magnitudes of the system parameters.

Our model can be extended in several directions. First, considering both upside risk and downside risk at the same time is a possible extension. This can be done by using the bi-directional option contract to hedge against both upside risk and downside risk. Second, in our paper we do not investigate the option writer's optimal behaviors, and thus one of the future study is to consider how the option writer's behavior and attitude affects the hedging behavior of the retailer. Third, as the tradable option and the real option provide different risk transfer mechanisms, we can further consider under a supply chain framework to see the equilibrium behavior between the supplier and the retailer. The research questions in this direction are as follows. Which mechanism is better? When will the tradable option be beneficial to the supplier? When will the supplier provide a real option? Are there certain contracts that can coordinate such a supply chain? All these issues should be addressed in future research.

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Appendix A

A.1. Downside risk measure: CVaR

We now formally describe the *CVaR* risk measure. *CVaR* is commonly used by financial institutions and companies involved in trading energy and other commodities. The original *CVaR* considers the loss to the decision maker. However, for our problem, it is more appropriate to consider the profit, which does not change the property of the risk measure as negative profit is loss (see Xu and Li, 2010). The objective to minimize loss-oriented *CVaR* becomes the objective to maximize profit-oriented *CVaR*. Given the newsvendor's random profit function $\hat{\Pi}(X;Q,K_q)$, we first define its value-at-risk (*VaR*), which is also called the η-quantile, where η is the confidence level with support over $(0,1]$, as follows:

$$\pi_\eta(\hat{\Pi}(X;Q,K_q)) = \inf\{Pr(\hat{\Pi}(X;Q,K_q) \le z) \ge \eta\}. \tag{9}$$

Given *VaR*, we can define *CVaR* in a general form for our model according to Rockafellar and Uryasev (2000) as follows:

$$CVaR_\eta(\hat{\Pi}(X;Q,K_q)) = E[\hat{\Pi}(X;Q,K_q)\,|\,\hat{\Pi}(X;Q,K_q) \le \pi_\eta(\hat{\Pi}(X;Q,K_q))]. \tag{10}$$

Now we can optimize the *CVaR* of the retailer's random profit by manipulating the decision variables. As this original definition is hard for optimization, we use the following equivalent definition, which is more convenient for optimization (see Rockafellar and Uryasev, 2000; Xu and Li, 2010):

$$\mathrm{CVaR}_\eta(\hat{\Pi}(X;Q,K_q)) = \max_{\xi\in\mathbb{R}}\{g(Q,K_q,\xi)\},$$

$$g(Q,K_q,\xi) := \xi - \frac{1}{\eta}\mathrm{E}[\xi - \hat{\Pi}(X;Q,K_q)]^+, \tag{11}$$

where $\mathbb{R}$ represents the real set and ξ is a variable in the real set. The confidence level η also reflects the newsvendor's preference for downside risk, i.e., the smaller the η is, the more risk averse is the newsvendor. When $\eta=1$, the newsvendor is risk neutral.

A.2. Proofs of the results

Before we prove the main results of this paper, we first find the optimal ξ, given Q and K_q in the *CVaR* formulation (11) to maximize $g(Q,K_q,\xi)$, i.e.,

$$\max_\xi\left\{\xi - \frac{1}{\eta}\left[\int_0^{K_q}(\xi-\hat{\pi}_1)^+ f(x)\,dx + \int_{K_q}^{Q}(\xi-\hat{\pi}_2)^+ f(x)\,dx + \int_Q^\infty (\xi-\hat{\pi}_3)^+ f(x)\,dx\right]\right\}. \tag{12}$$

Given K_q, when $K_q \ge Q$, we can replicate the analysis in Xu and Li (2010) to obtain the optimal Q^* as

$$Q^* = \min\left\{\frac{1}{s+b-K_p}\left[(s-K_p)F^{-1}\left(\frac{\eta(s+b-c)}{s+b-K_p}\right) + bF^{-1}\left(1-\frac{\eta(c-K_p)}{s+b-K_p}\right)\right], K_q\right\}, \tag{13}$$

where $K_p \le c$. $Q^* = K_q$ when $K_p > c$.

Thus, we first consider the case where $Q \ge K_q$ in this part. To ease the notation, we use $g(\xi)$ to denote $g(Q,K_q,\xi)$. Then, given K_q and Q, depending on the value of ξ, we can give the expression of $g(\xi)$, and then find the optimal unconstrained solution to it, by considering the following four cases:

CASE I: When $\xi \le (K_p-v)K_q+(v-c)Q-P(K_q)$, we have

$$g(\xi) = \xi - \frac{1}{\eta}\int_{((s-c+b)Q-P(K_q)-\xi)/b}^{\infty}[\xi-(s-c+b)Q+P(K_q)+bx]f(x)\,dx.$$

Then, for the unconstrained version of the optimization problem (12), the optimal ξ satisfies the first-order condition and we have $\bar{\xi}(Q,K_q) = (s-c+b)Q-P(K_q)-bF^{-1}(1-\eta)$.

CASE II: When $(K_p-v)K_q+(v-c)Q-P(K_q) < \xi \le (v-c)Q+(s-v)K_q-P(K_q)$, we have

$$g(\xi) = \xi - \frac{1}{\eta}\int_{((s-c+b)Q-P(K_q)-\xi)/b}^{\infty}[\xi-(s-c+b)Q+P(K_q)+bx]f(x)\,dx - \frac{1}{\eta}\int_0^{(\xi-(K_p-v)K_q-(v-c)Q+P(K_q))/(s-K_p)}[\xi-(K_p-v)K_q-(v-c)Q + P(K_q)-(s-K_p)x]f(x)\,dx.$$

Then, the first-order condition of the unconstrained version of (12) implies that $\hat{\xi}(Q,K_q)$ satisfies

$$\eta - \left(1-F\left(\frac{(s-c+b)Q-P(K_q)-\xi}{b}\right)\right) - F\left(\frac{\xi-(K_p-v)K_q-(v-c)Q+P(K_q)}{s-K_p}\right) = 0. \tag{14}$$

CASE III: When $(v-c)Q+(s-v)K_q-P(K_q) < \xi \le (s-c)Q-P(K_q)$, we have

$$g(\xi) = \xi - \frac{1}{\eta}\int_0^{K_q}[\xi-(K_p-v)K_q-(v-c)Q+P(K_q)-(s-K_p)x]f(x)\,dx - \frac{1}{\eta}\int_{K_q}^{(\xi-(v-c)Q+P(K_q))/(s-v)}[\xi-(v-c)Q+P(K_q)-(s-v)x]f(x)\,dx - \frac{1}{\eta}\int_{((s-c+b)Q-P(K_q)-\xi)/b}^{\infty}[\xi-(s-c+b)Q+P(K_q)+bx]f(x)\,dx.$$

Then, the first-order condition of the unconstrained version of (12) implies that $\tilde{\xi}(Q,K_q)$ satisfies

$$\eta - \left(1-F\left(\frac{(s-c+b)Q-P(K_q)-\xi}{b}\right)\right) - F\left(\frac{\xi-(v-c)Q+P(K_q)}{s-v}\right) = 0. \tag{15}$$

CASE IV: When $\xi > (s-c)Q-P(K_q)$, we have

$$g(\xi) = \xi - \frac{1}{\eta}\int_0^{K_q}[\xi-(K_p-v)K_q-(v-c)Q+P(K_q)-(s-K_p)x]f(x)\,dx - \frac{1}{\eta}\int_{K_q}^{Q}[\xi-(v-c)Q+P(K_q)-(s-v)x]f(x)\,dx - \frac{1}{\eta}\int_Q^\infty[\xi-(s-c+b)Q+P(K_q)+bx]f(x)\,dx.$$

The derivation of $g(\xi)$ with respect to ξ is $1-(1/\eta)$, which is negative as $\eta \le 1$, so $g(\xi)$ is decreasing in ξ.

By summarizing the above four cases, we conclude that the optimal ξ for the constrained problem (12) is

$$\xi^* = \begin{cases} \overline{\xi} & \text{if } (s+b-v)Q-(K_p-v)K_q \le bF^{-1}(1-\eta), \\ \hat{\xi} & \text{if } bF^{-1}(1+F(K_q)-\eta)+(s-v)K_q \ge (s+b-v)Q > bF^{-1}(1-\eta)+(K_p-v)K_q, \\ \tilde{\xi} & \text{otherwise.} \end{cases}$$

Here we implicity assume $F(K_q) \le \eta$. When $F(K_q) > \eta$, similar to the above analysis, we can conclude that there are only the first two cases.

Proof of Theorem 4.1. Now, given K_q, we start with determining the optimal Q when $Q \ge K_q$ to maximize

$$\max_{Q \ge K_q} g(\xi^*). \tag{16}$$

This will be achieved by considering the following three cases and under the assumption that $F(K_q) < \eta$. The case where $F(K_q) > \eta$ is similar and is neglected in this part.

Case 1: When $(s+b-v)Q-(K_p-v)K_q \le bF^{-1}(1-\eta)$, the optimal solution $\xi^* = \overline{\xi}(Q)$. Substituting ξ^* into $g(\xi)$ and taking derivative with respect to Q, we get

$$\frac{\partial g(\overline{\xi})}{\partial Q} = \frac{1}{\eta}\int_{((s-c+b)Q-P(K_q)-\overline{\xi})/b}^{\infty} (s-c+b)f(x)\,dx > 0.$$

Case 2: When $bF^{-1}(1+F(K_q)-\eta)+(s-v)K_q \ge (s+b-v)Q > bF^{-1}(1-\eta)+(K_p-v)K_q$,
the optimal solution $\xi^* = \hat{\xi}$. Substituting ξ^* into $g(\xi)$ and taking derivative with respect to Q, we get

$$\frac{\partial g(\hat{\xi})}{\partial Q} = \frac{1}{\eta}\int_{((s-c+b)Q-P(K_q)-\hat{\xi})/b}^{\infty} (s-c+b)f(x)\,dx + \frac{1}{\eta}\int_0^{(\hat{\xi}-(K_p-v)K_q-(v-c)Q+P(K_q))/(s-K_p)} (v-c)f(x)\,dx.$$

Combining the above equation with (14) and equating it to zero, we find

$$F\left(\frac{(s-c+b)Q-P(K_q)-\xi}{b}\right) = 1-\eta\frac{c-v}{s+b-v}$$
$$F\left(\frac{\xi-(K_p-v)K_q-(v-c)Q+P(K_q)}{s-K_p}\right) = \eta\frac{s+b-c}{s+b-v},$$

so the optimal Q_2^* to the unconstrained version of (16) satisfies

$$(s-v+b)Q_2^*-(K_p-v)K_q = bF^{-1}\left(1-\frac{\eta(c-v)}{s+b-v}\right)+(s-K_p)F^{-1}\left(\frac{\eta(s+b-c)}{s+b-v}\right).$$

We need further to check whether Q_2^* satisfies the constraint on Q. It is straightforward that $(s+b-v)Q_2^* > bF^{-1}(1-\eta)+(K_p-v)K_q$, so we only need to consider the following three cases:

(i) if $K_q < F^{-1}(\eta(s+b-c)/(s+b-v))$, we have

$$\begin{aligned} &bF^{-1}(1+F(K_q)-\eta)+(s-v)K_q-(s+b-v)Q_2^* \\ &= bF^{-1}(1+F(K_q)-\eta)+(s-K_p)K_q+(K_p-v)K_q-(s-v+b)Q_2^* \\ &< b\left(F^{-1}(1+F(K_q)-\eta)-F^{-1}\left(1-\frac{\eta(c-v)}{s+b-v}\right)\right)+(s-K_p) \\ &\quad\times\left(K-F^{-1}\left(\frac{\eta(s+b-c)}{s+b-v}\right)\right) < 0. \end{aligned}$$

Thus, $bF^{-1}(1+F(K_q)-\eta)+(s-v)K_q < (s+b-v)Q_2^*$, which implies that the optimal Q_2^* does not lie within this range.

(ii) if

$$K_q > K^M = \frac{b}{s+b-K_p}F^{-1}\left(1-\frac{\eta(c-v)}{s+b-v}\right)+\frac{(s-K_p)}{s+b-K_p}F^{-1}\left(\frac{\eta(s+b-c)}{s+b-v}\right),$$

we find $Q_2^* < K_q$, which implies that $Q^A = K_q$.

(iii) otherwise, $Q^A = Q_2^*$.

Case 3: When $bF^{-1}(1+F(K_q)-\eta)+(s-v)K_q < (s+b-v)Q$, the optimal solution is $\xi^* = \tilde{\xi}$. Substituting ξ^* into $g(\xi)$ and taking derivative with respect to Q, we get

$$\frac{\partial g(\xi)}{\partial Q} = \frac{1}{\eta}\int_0^{(\xi-(v-c)Q+P(K_q))/(s-v)} (v-c)f(x)\,dx + \frac{1}{\eta}\int_{((s-c+b)Q-P(K_q)-\xi)/b}^{\infty} (s-c+b)f(x)\,dx.$$

Combining the above condition with (15) and equating it to zero, we find that the optimal Q_3^* to the unconstrained version of (16) satisfies

$$(s-v+b)Q_3^* = bF^{-1}\left(1-\frac{\eta(c-v)}{s+b-v}\right)+(s-v)F^{-1}\left(\frac{\eta(s+b-c)}{s+b-v}\right).$$

Similar to Case 2, we can show that the optimal Q_3^* lies within this range, i.e., $Q^A = Q_3^*$, as $K_q \le F^{-1}(\eta(s+b-c)/(s+b-v))$. Furthermore, in this case Q_3^* is always larger than K_q.

Summarizing the above three cases, and combining it with the case where $Q < K_q$ analyzed before, we obtain the corresponding results. □

Proof of Lemma 4.5. We only consider part (i) of this lemma and other parts can be similarly proved. We divide the proof into two parts: (a) when $Q_1 < Q^u$, then $K^M < Q_2 < Q^u$; (b) when $Q_2 < Q^u$ then, $Q_1 < Q^u$. It is obvious that $G(t)$ is increasing in t, which is a useful characteristic that we use in the following proof.

(a) When $Q_1 < Q^u$, we find

$$\begin{aligned} G(Q^u) &= (s+b-K_p)Q^u - \left[bF^{-1}\left(1-\frac{\eta(c-v)}{s+b-v}\right)+(s-K_p)F^{-1}\left(\frac{\eta(s+b-c)}{s+b-v}\right)\right] = (s+b-v)Q^u - \left[bF^{-1}\left(1-\frac{\eta(c-v)}{s+b-v}\right)\right. \\ &\quad \left.+(s-K_p)F^{-1}\left(\frac{\eta(s+b-c)}{s+b-v}\right)+(K_p-v)F^{-1}\left(\frac{s+b-c}{s+b-v}\right)\right] \\ &= (s+b-v)(Q^u-Q_1^*) \ge 0 \end{aligned}$$

As $G(Q_2)=0$ and $G(Q)$ is increasing in Q, we get $Q_2 \le Q^u$. Moreover, we have

$$\begin{aligned} (s+b-K_p)Q_2 &= bF^{-1}\left(1-\eta\frac{(K_p-v)F(Q_2)-(K_p-c)}{s+b-K_p}\right) \\ &\quad +(s-K_p)F^{-1}\left(\eta\frac{(s+b-c)-(K_p-v)F(Q_2^*)}{s+b-K_p}\right) \\ &\ge bF^{-1}\left(1-\eta\frac{(K_p-v)F(Q^u)-(K_p-c)}{s+b-K_p}\right)+(s-K_p) \\ &\quad \times F^{-1}\left(\eta\frac{(s+b-c)-(K_p-v)F(Q^u)}{s+b-K_p}\right) \\ &= bF^{-1}\left(1-\frac{\eta(c-v)}{s+b-v}\right)+(s-K_p)F^{-1}\left(\frac{\eta(s+b-c)}{s+b-v}\right) \\ &= K^M, \end{aligned}$$

in which the inequality holds as the left side of the equation is decreasing in Q.

(b) When $Q_2 < Q^u$, we have $G(Q^u) > 0$ as $G(Q_2)=0$ and $G(t)$ is an increasing function. That is,

$$G(Q^u) = (s+b-K_p)Q^u - \left[bF^{-1}\left(1-\frac{\eta(c-v)}{s+b-v}\right)+(s-K_p)F^{-1}\left(\frac{\eta(s+b-c)}{s+b-v}\right)\right] > 0.$$

Thus, $Q_1 < Q^u$.

Combining these two parts, we get the part (i) of this lemma. □

Proof of Theorem 4.2. The idea of this part is to derive the optimal K_q^*, given the optimal Q^* established in Theorem 4.1. We only prove the case where $K_p \ge c$ here; when $K_p < c$, the proof is similar and we omit it here. Then the objective is to solve the following problem:

$$\max_{K_q} g(\xi^*), \tag{17}$$

given the results in Theorem 4.1. We analyze the following three cases, depending on the ranges of Q, K_q, and the corresponding ξ^*:

Case 1: When $K_q \ge K^M$, the optimal $Q^A = K_q$ by Theorem 4.1. Substituting $Q = K_q$ into $g(\xi)$ and taking derivative with respect to K_q, we get

$$\frac{\partial g(\xi)}{\partial K_q} = -\frac{1}{\eta}\int_{((s-c+b)K_q - P(K_q)-\xi)/b}^{\infty} [-(s-c+b)+(K_p-v)F(K_q)]f(x)\,dx$$
$$-\frac{1}{\eta}\int_0^{(\xi-(K_p-c)K_q+P(K_q))/(s-K_p)} [-(K_p-c)+(K_p-v)F(K_q)]f(x)\,dx,$$

which is equivalent to

$$\eta\frac{\partial g(\xi)}{\partial K_q} = ((s-c+b)-(K_p-v)F(K_q))\left(1-F\left(\frac{(s-c+b)K_q-P(K_q)-\xi}{b}\right)\right)$$
$$+((K_p-c)-(K_p-v)F(K_q))F\left(\frac{\xi-(K_p-c)K_q+P(K_q)}{s-K_p}\right).$$

Combining this with (15), we get

$$\eta\frac{\partial g(\xi)}{\partial K_q} = -\eta[(K_p-v)F(K_q)-(s+b-c)]-(s+b-K_p)$$
$$\times F\left(\frac{\xi-(K_p-c)K_q+P(K_q)}{s-K_p}\right)$$
$$= -\eta[(K_p-v)F(K_q)-(K_p-c)]+(s+b-K_p)-(s+b-K_p)$$
$$\times F\left(\frac{(s-c+b)Q-P(K_q)-\xi}{b}\right).$$

As $F((\xi-(K_p-c)K_q+P(K_q))/(s-K_p)) \in [0,\eta]$, for $K_q \le F^{-1}(K_p-c/K_p-v)$, $g(\xi)$ is increasing in K_q; and for $K_q \ge F^{-1}((s+b-c)/(K_p-v))$, $g(\xi)$ is decreasing in K_q. So the optimal K_q^* should lie in $[F^{-1}((K_p-c)/(K_p-v)), F^{-1}((s+b-c)/(K_p-v))]$ and satisfy

$$(s+b-K_p)K_q = bF^{-1}\left(1-\eta\frac{(K_p-v)F(K_q)-(K_p-c)}{s+b-K_p}\right)+(s-K_p)$$
$$\times F^{-1}\left(\eta\frac{(s+b-c)-(K_p-v)F(K_q)}{s+b-K_p}\right),$$

which implies that both the optimal order quantity and optimal hedging quantity are Q_2.

Case 2: When $\underline{K} \le K_q < K^M$, $Q^A = 1/(s+b-v)[(K_p-v)K_q+(s+b-K_p)K^M]$. Substituting Q^A into $g(\xi)$ and taking derivative with respect to K_q, we get

$$\frac{\partial g(\xi)}{\partial K_q} = -\frac{1}{\eta}\int_{((s-c+b)Q-P(K_q)-\xi)/b}^{\infty}[(K_p-v)F(K_q)]f(x)\,dx$$
$$-\frac{1}{\eta}\int_0^{(\xi-(K_p-v)K_q-(v-c)Q+P(K_q))/(s-K_p)}[-(K_p-v)+(K_p-v)$$
$$\times F(K_q)]f(x)\,dx,$$

which is equivalent to

$$\eta\frac{\partial g(\xi)}{\partial K_q} = (-(K_p-v)F(K_q))\left(1-F\left(\frac{(s-c+b)Q-P(K_q)-\xi}{b}\right)\right)$$
$$+((K_p-v)-(K_p-v)F(K_q))F\left(\frac{\xi-(K_p-v)K_q-(v-c)Q+P(K_q)}{s-K_p}\right).$$

Combining this with (14), we get

$$F\left(\frac{\xi-(K_p-v)K_q-(v-c)Q+P(K_q)}{s-K_p}\right) = \eta F(K_q).$$

Moreover, from Theorem 4.1, we know

$$F\left(\frac{\xi-(K_p-v)K_q-(v-c)Q+P(K_q)}{s-K_p}\right) = \eta\frac{s+b-c}{s+b-v},$$

so we get that the optimal K_q^* satisfies $F(K_q^*) = F(Q^u)$, i.e., $K_q^* = Q^u$.

Case 3: When $K_q < \underline{K}$, $Q^A = Q^N$, which is independent of K_q. Substituting Q^A into $g(\xi)$ and taking derivative with respect to K_q, we get

$$\frac{\partial g(\xi)}{\partial K_q} = -\frac{1}{\eta}\int_{((s-c+b)Q-P(K_q)-\xi)/b}^{\infty}[(K_p-v)F(K_q)]f(x)\,dx - \frac{1}{\eta}\int_0^{K_q}$$
$$-(K_p-v)f(x)\,dx\frac{1}{\eta}\int_0^{(\xi-(K_p-v)K_q-(v-c)Q+P(K_q))/(s-K_p)}$$
$$\times[(K_p-v)F(K_q)]f(x)\,dx.$$

Combining this with (14), we get

$$\frac{\partial g(\xi)}{\partial K_q} = -\left(1-\frac{1}{\eta}\right)F(K_q) > 0.$$

Thus, the value function is increasing in K_q and the optimal K_q does not lie within $[0, \underline{K})$.

Summarizing the above three cases, and with the results of Lemma 4.5, we obtain the corresponding results. □

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